Online supplementary information for "A vortex plasma model analysis of the THz conductivity and diamagnetism in $La_{2-x}Sr_xCuO_4$"

Time-domain THz spectroscopy - We have measured the THz range optical conductivity using a home-built transmission based time-domain THz spectrometer. In this technique, we split a femtosecond laser pulse along two paths and sequentially excite a pair of photoconductive 'Auston'-switch antennae on radiation damaged silicon on sapphire. A broadband THz range pulse is emitted by one antenna, transmitted through the LSCO film, and measured at the other antenna. By varying the length-difference of the two paths, the electric field of the transmitted pulse is measured as a function of time. Ratioing the Fourier transform of the transmission through the LSCO film on a substrate to that of a bare reference substrate, we resolve the frequency dependent complex transmission. The transmission is inverted to obtain the complex conductivity by the standard formula for thin films on a substrate: $\tilde{T}(\omega) = [(1+n)/(1+n+Z_0\tilde{\sigma}(\omega)d)]e^{i\Phi_s}$ where Φ_s is the phase accumulated from the small difference in thickness between the sample and reference substrates and n is the substrate index of refraction.

Scaling analysis of the fluctuation conductivity - In previous work [1] we have also performed a scaling analysis that allowed us to extract out a characteristic fluctuation rate of the superconductivity. This analysis follows from the fact that for a fluctuating superconductor one expects that the relation

$$\sigma_S(\omega) = \frac{G_Q}{t}\frac{k_B T_\theta^0}{\hbar\Omega}\mathcal{S}(\frac{\omega}{\Omega}) \qquad (1)$$

holds for the portion of the conductivity, σ_S, due to superconducting fluctuations. Here $G_Q = e^2/\hbar$ is the quantum of conductance, t is the inter-CuO_2 plane spacing, T_ϕ^0 is a temperature dependent prefactor and Ω is the characteristic fluctuation rate. This scaling function is similar to the one proposed by Fisher, Fisher, and Huse [2] and is identical to the one used in previous THz measurements on underdoped BSCCO [3]. In Fig. 4a we show the collapsed phase $\varphi = \tan^{-1}\sigma_2/\sigma_1$ from the data in Fig. 1 as a function of reduced frequency ω/Ω at temperatures from 22 K to 30 K. The phase is an increasing function of ω/Ω, with the metallic limit $\varphi = 0$ reached at $\omega/\Omega \to 0$ and φ becoming large (but bounded by $\pi/2$) as $\omega/\Omega \to \infty$. We plot the extracted fluctuation rate Ω as a function of temperature in Fig. 4b. As noted previously [1], we continue to obtain good scaling and data collapse if we push the analysis 3-4 K above the temperature of the obvious onset in σ_2. This region shows a linear dependence of Ω on temperature.

In the main text, we use $\Omega(T)$ to evaluate the magnitude of the fluctuation contribution to the conductivity. We evaluate the magnitude at a frequency of fixed

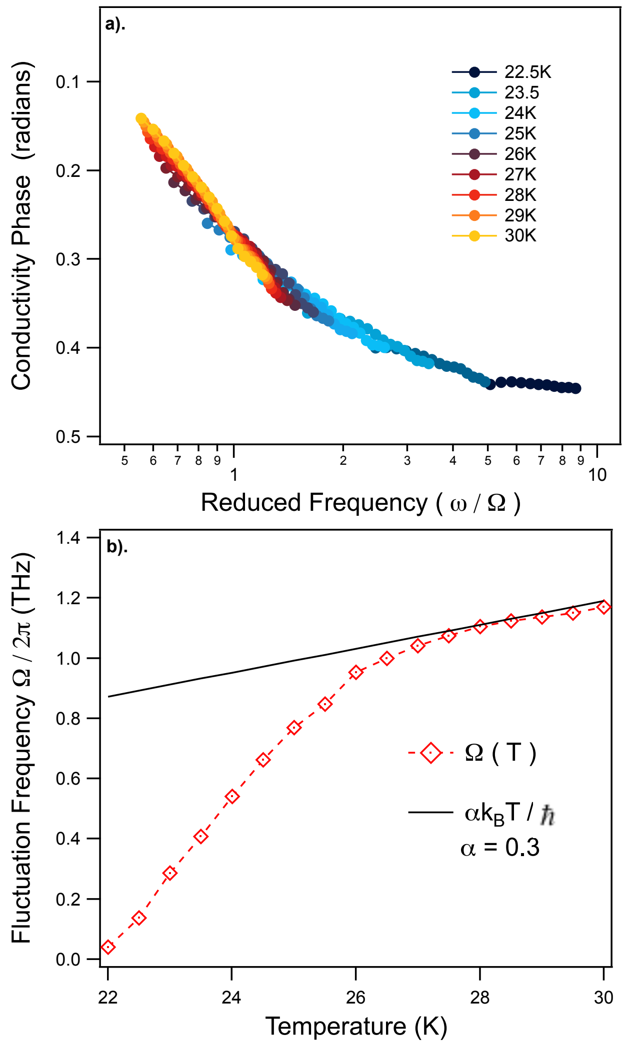

FIG. 1: (a) The conductivity phase $\varphi = \tan^{-1}\sigma_2/\sigma_1$ vs. scaled frequency ω/Ω as described in the text. (b) The extracted fluctuation rate Ω obtained from the proportionality of the collapsed phase and the scaling function $S(\omega/\Omega)$.

proportionality of 1/2 the characteristic fluctuation frequency at each temperature, i.e. $\sigma_S(\omega = \Omega(T)/2, T)$. The S function in Eq. 4 then becomes the same constant of order unity for all temperatures; in the analysis in this paper we set it equal to one.

Modeling of normal state conductivity - We estimate the normal state contribution to the conductivity by using the Drude model $\sigma = \frac{\omega_p^2}{4\pi}\frac{\tau}{1-i\omega\tau}$, which should be valid in the cuprate normal state at low enough frequencies. In this model note that $\sigma_2(\omega)/\sigma_1(\omega) = \omega\tau$. Assuming τ is relatively frequency independent in this frequency range, we extract τ at high temperatures by finding the slope of $\frac{\sigma_2}{\sigma_1}(\omega)$. Using the high temperature τ and $|\sigma|$, we find the plasma frequency ω_p. We then fit $1/\tau$ to a power law form $a_1{+}b_1T^n$ and ω_p to a linear form $a_2{+}b_2T$ at high temperature and extrapolate these fits to low temperature. Since $\omega\tau << 1$ in this frequency regime, the normal state background contribution is well approximated as $|\sigma_{bg}| = \omega_p^2\tau/4\pi$. The error bars on the vortex diffusion and fluctuation conductivity come from our uncertainty in fitting $1/\tau$ and ω_p in the high temperature range. The upper limit of $|\sigma_{bg}|$ was set with the values of the $1/\tau$ fitting parameters (in THz units) of a_1=4.08, b_1=1.33×10^{-6}, and n=2.82; the lower limit of $|\sigma_{bg}|$ used the values a_1=3.72, b_1=2×10^{-4}, and n=1.85. In fitting ω_p we allowed a_2 to have a very small frequency dependence (varying by $\approx 2\%$ of the average value of 4.5 GHz), while b_2 was kept constant at b_2=-1.09 MHz/K.

Molecular beam epitaxy of $La_{1.905}Sr_{0.095}CuO_4$ films - The LSCO films were deposited on 1-mm-thick single-crystal $LaSrAlO_4$ substrates, epitaxially polished perpendicular to the (001) direction, by atomic-layer-by-layer molecular-beam-epitaxy (ALL-MBE) [4]. The samples were characterized by reflection high-energy electron diffraction, atomic force microscopy, X-ray diffraction, and resistivity and magnetization measurements, all of which indicate excellent film quality. The thickness is known accurately by counting atomic layers and RHEED oscillations, as well as from so-called Kiessig fringes in small-angle X-ray reflectance and from finite thickness oscillations observed in XRD pattern. For further details see Refs. [1, 4].

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